1. Research question:

What is the relationship between temperature (K) and spring constant (k) of a rubber band?

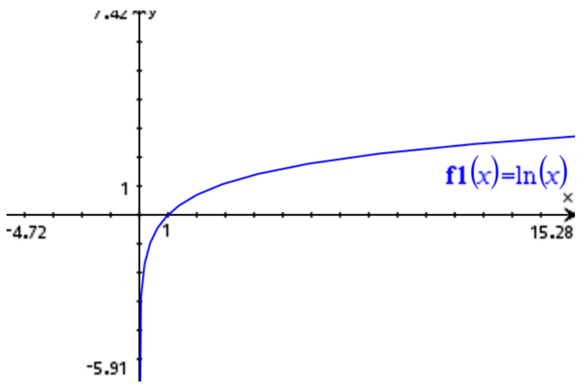
2. Background information:

This report investigates the relationship between the temperature of a rubber band and the spring constant of a rubber band, and how this varies across temperatures. In order to understand this experiment, we first need to understand the fundamental force behind polymeric elasticity. A rubber band is composed of numerous, interlinked, disordered polymer chains. When the band is stretched, the chains are aligned in a predictable order, decreasing entropy. This is described by the equation

$$S = Kb\ ln\Omega$$

Where S = entropy, Kb = Boltzmann's constant, and Ω = the number of possible microstates.

As the band is stretched, the number of possible microstates is decreased greatly. Looking at the graph of the natural logarithm function:



We can see that as microstates reduce, entropy reduces as well (as there can never be less than one microstate, a system can never have negative entropy, which makes sense)
As in every process, the total entropy of an isolated system must increase or stay the same (second law of thermodynamics), this gives rise to the elastic restoring force, which attempts to return the system to its original, higher entropy state. Specifically, the random oscillations caused by temperature cause the state of the system to evolve constantly, and these evolutions tend towards the highest entropy state, where the polymers are randomly oriented (unstretched band).

Heating up the rubber band increases the initial random kinetic energy of the particles, which increases its entropy in the unstretched state. Therefore, when it is stretched, there is a greater difference between the entropy of the ordered (stretched) state, and the original, disordered (unstretched) state, which increases the elastic restoring force, and therefore the stiffness of the band. This relationship be seen in the equation

$$k = \frac{3KbT}{Nl^2}$$

where k = spring constant, Kb = Boltzmann's constant, T = absolute temperature (kelvin), N = number of monomer units in the polymer chain, and l = length of each monomer segment.

Understanding the derivation of this equation is beyond the scope of this investigation, as it comes from entropic spring theory and polymer physics, however the notable aspects of it are that it suggests that spring constant (k) is proportional to temperature (t), as the other variables are constants for the same spring.
From the syllabus, we are given:

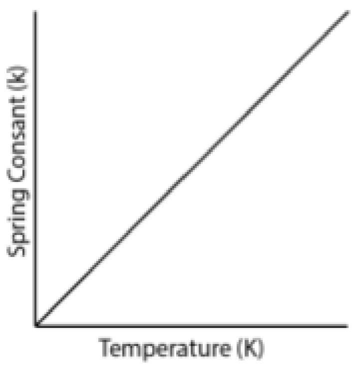
$$F = \quad - kx$$

where F = force required to displace the band (in newtons), k = spring constant, and x = displacement of the band (amount of stretch). While this is a simplification, and young's modulus is more accurate, this is sufficient, as we are staying within the elastic limit of the spring, where the relationship is mostly linear. This equation can be algebraically rearranged to

$$k = |x|/F$$

To summarize, my experimental hypothesis is that k will be proportional to temp, and so will F. This means that graphing k against T should yield a graph with a constant gradient:

Figure 1 - Spring constant vs Temperature



Please note T=0 (absolute zero) is impossible, and other forces would become dominant, leading to a 0,0 intercept not being realistic.

Figure 1 describes the relationship between K and T at any given range, as long as this range is not close to T=0, where other forces become dominant. However, the elastic restoring force due to entropy would be 0 at T=0 as there would be no random kinetic motion to make the system evolve.

3. Variables:

3.1 Independent variable

My independent variable (IV) is the temperature of the rubber band in Kelvin, as measured using a digital thermometer. I've opted for the intervals used in the table below as they're the highest temperatures I could reach using the school-provided kettle and the lowest temperatures I could reach by allowing natural cooling and adding ice where required. I've opted for a digital thermometer as it allows me to minimise uncertainties in my experimental data.

3.2 Dependent variable:

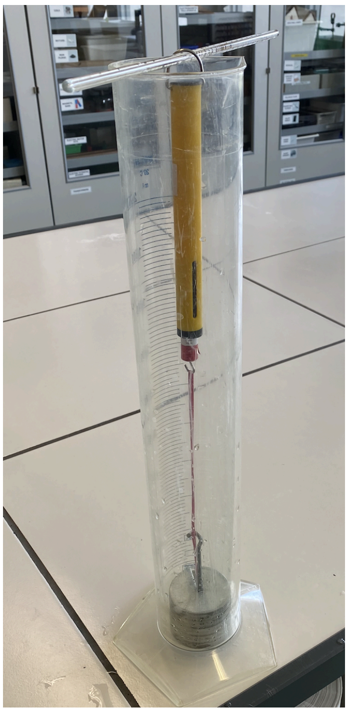
My dependent variable (DV) is the force (in newtons) taken to extend a rubber band by 21cm. I chose this DV as in my pilot study, I discovered that because of how minimal the force changes were, and how low the spring constant of the newtonmeter (connected in series) was compared to that of the rubber band, there was a negligible change in length. Therefore, for measuring spring constant, it was most appropriate to measure force required to stretch the band a standard amount. I utilised the most sensitive newtonmeter available at our school, which was an analogue newtonmeter that has intervals of 0.05N and a maximum capacity of 3N.

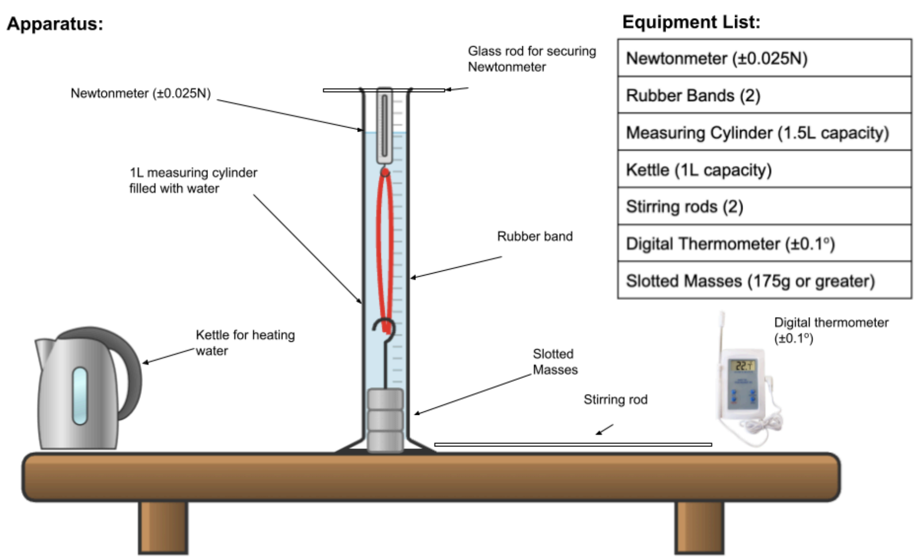
3.3 Controls

Control variable	Discussion
Verticality of rubber band	If the rubber band was stretched at different angles for each trial, this might significantly impact the force measured, which would be significant since the forces being observed are so small. This would lead to inaccurate results, and create noise in the observed data. Therefore, the newtonmeter anchor will always be placed directly above the anchor of the rubber band using a stirring rod placed across the diameter of the measuring cylinder.
Temperature of the spring	The temperature of the water (and therefore the spring) must be controlled and measured in order to ensure that there is high precision in the data and that readings across trials can be compared. Additionally, since the water is cooling due to air and/or ice, it is essential to stir the cylinder occasionally in order to ensure even heat distribution and an accurate reading.
Volume of water	The volume of water must be controlled in order to minimise fluctuations in water pressure
Displacement of the rubber band	The rubber band must be equally lengthened across trials, in order to ensure that the spring constant can be accurately derived.
Equipment used - Rubber Band - Newtonmeter - Cylinders - Thermometer	I needed to control the equipment used for each of the trials for a variety of reasons. Some of the reasons were - Different rubber bands have different numbers of polymers and spring constants, which would influence results seen - Different newtonmeters have different systemic errors - Different thermometers have different random errors.

4. Methodology:

4.1 equipment/diagram





4.2 preliminary testing

Aspect of experiment	Rationale
Rubber band used	After experimenting with a range of rubber bands, the rubber band chosen was decided upon as it is thick enough to loop onto the newtonmeter, and elastically deforms over a wide range of lengths, meaning that Hooke's law is a valid approximation for this band, as the deformation is mostly elastic and not plastic.
Number of rubber bands used	I used two rubber bands in parallel instead of one, as this made the forces larger, allowing me to measure them more accurately. This does mean I have to divide the experimental spring constant by two in order to get the spring constant of one of the bands, but functionally they behave as if they were one band, so this should not influence the results of my experiments.
Submersion of rubber in water	After attempting the experiment by submerging the rubber in water and then testing it, I discovered that the rubber cooled far too rapidly for the experiment to work. Therefore, I opted to conduct the entire experiment underwater. Because the specific heat capacity of rubber is so low, the temperature of the water is equal to the temperature of the rubber band if it is submerged. This also makes controlling the temperature of the band much easier.
Vertical setup of experiment	At first I attempted to set up the experiment horizontally, which would have allowed me to conduct the experiment in a temperature-controlled water bath. However, the bath was too narrow for the experimental apparatus and it was difficult to properly anchor the rubber band to the bath in a reversible manner. While superglue would have worked, it would also have permanently damaged the equipment.
Choice of rubber as experimental material	It was unfeasible to use springs, as the energy required to heat them up to a temperature where a difference in spring constant could be observed was way too high to realistically achieve in a high school context.
Controlling length of rubber band	During the pilot study, it was evident that because of the difference in spring constant between the newtonmeter and the bands length of the bands did not change notably across trials. I measured it repeatedly at different temperatures multiple times, and there was no notable change, allowing me to assume it remains mostly constant for the real experiment.
Range of temperatures chosen.	20-80^{o} was chosen as this range was the largest accessible using the equipment we had readily available, and from the pilot study it was discovered that this range was sufficient to observe changes
Amount of weight chosen for anchoring	After testing repeatedly with the rubber bands chosen, this weight was decided upon as it was large enough to anchor the rubber band strongly.
Jiggling of newtonmeter	It was found that the newtonmeter would get stuck at certain values as the mechanism was not adequately lubricated. This problem could be mitigated by jiggling the newtonmeter up and down before recording values.

4.3 methodology

Operationalized Procedure:

1. Take out equipment and place on table
2. Fill kettle to line marked “Max”, and set to boil
3. Thread bands through slotted mass hook on one end, and newtonmeter on other
4. Place slotted masses inside cylinder
5. Thread newtonmeter through stirring rod above cylinder, ensure it is centered above masses
6. Wait for water to boil, then pour into measuring cylinder
7. Wait for water to reach 80^o, gently stir water to ensure even heat distribution, jiggle around newtonmeter and then record value in table
8. Repeat step (7). at intervals of 10^o until 20^o. Ice may be used to speed up cooling, but excess water must be removed to ensure volume remains constant.
9. Repeat steps (1-7) 6 more times, and continue recording data in the table.

4.4 Accuracy and reliability of the experiment

This experiment is likely highly precise. This is due to the low equipment uncertainties and the use of digital testing equipment to record the DV. There is little room for human error or random error, meaning the experiment will likely yield highly precise results, with a strong correlation coefficient. In terms of the accuracy of the results: as discussed in the control variables, all extraneous variables that could have adverse effects on the outcome of the experiment were mostly controlled. This means the experiment will also likely have high accuracy, as the methodology seems to have high construct validity.

5. Safety

The most pressing safety concerns with this experiment come from the handling of hot water. Take care and move slowly, and ensure that there are no spills or contact with heated elements. Furthermore, do not spill water around the kettle or the socket in which it is plugged, as this poses additional risks. Keep the rubber bands away from your eyes, and wear protective gloves should they not impede dexterity.

6. Raw data:

6.1 raw data tables

Temperature of the band (Celsius)	Force required to displace the band ~21cm (Newtons)						
	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Trial 6	Trial 7
80	1.40	1.50	1.35	1.50	1.50	1.40	1.50
70	1.35	1.45	1.30	1.45	1.45	1.35	1.45
60	1.30	1.40	1.25	1.35	1.40	1.30	1.40
50	1.25	1.35	1.20	1.35	1.35	1.25	1.35
40	1.20	1.30	1.15	1.30	1.30	1.20	1.30
30	1.15	1.25	1.10	1.25	1.25	1.15	1.25
20	1.10	1.20	1.05	1.20	1.20	1.10	1.20

6.2 Initial observations

The results appear to be quite consistent. There is a fairly consistent trend in decreasing netometer values across trials. It's interesting that while the individual magnitude of this change might vary across data points, it's remarkably consistent across trials and across larger temperature ranges. This suggests that while small instrumental errors hampered precision, the experiment was in fact still precise on larger scales. A notable outlier was trial 3, where the recorded force was significantly lower across all trials. This is likely due to a setup error causing systematic error for that trial.

7. Processed data

7.1 Processed data table

Temperature (Kelvin)	Mean force required to displace 21cm	Percentage uncertainty for force required (closest integer)	Average spring constant	Percentage uncertainty of spring constant
353	1.45	5	6.90	5.17
343	1.40	5	6.67	5.36
333	1.34 (rounded to 1.35)	6	6.39	5.59
323	1.30	6	6.19	5.77
313	1.25	6	5.95	6.00
303	1.20	6	5.71	6.25
293	1.15	7	5.48	6.52

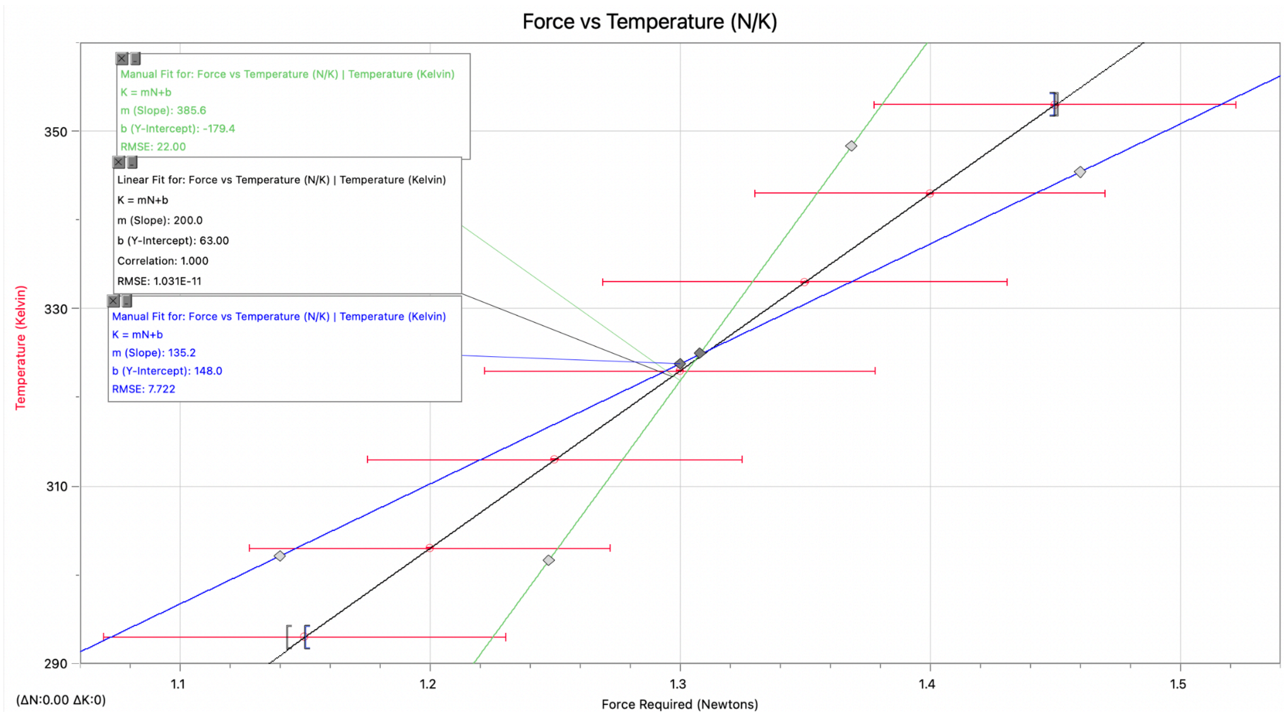
7.2 calculations

Process	General	Example
Celsius to Kelvin Conversion	*celsius value* + 273	80 + 273 = 353K
Mean force required to displace 21cm	$\frac{\sum_{F=0}^{n} x_F}{n}$ where n = number of trials, x_F = force for trial F.	(1.40+1.50+1.35+1.50+1.50+ 1.40+1.50)/7 = 1.45
Percentage Uncertainty (Using range/2)	$\frac{\frac{max-min}{2}}{mean\ value} \times 100$	(1.5-1.35)/2 = 0.075 (0.075/1.45)100 = 5.17 (rounded to integer is 5%)
Percentage Uncertainty (using equipment error)	*Smallest increment*/2	0.05/2 = 0.025 (range/2 is larger, so that was used)
Average Spring Constant	f = kx k = f/x given in formula booklet	1.45/0.021 = 6.904 (to 3sf is 6.90)

7.3 discussion of uncertainties.

Uncertainties were introduced to my experiment through a variety of sources, however the two most prominent were likely equipment uncertainty: the newtonmeter recorded to an accuracy of 0.05N, meaning measurements to lower than that were not possible, therefore I have rounded my calculated values so that they are no more precise than this. The second source of uncertainty is difficult to quantify: there could have been subtle differences in setup and human error introduced while reading the newtonmeter. Therefore, I have utilised range/2 in order to calculate uncertainty using the values, and employed that as it was higher than the equipment uncertainty. This does mean that the uncertainty of the experiment is high, but this is a limitation of the equipment available.

8. Graphed Data



9. Conclusion

The aim of this investigation was to determine the relationship between the temperature of a rubber band (in Kelvin) and its spring constant (k). The processed data clearly demonstrates a positive correlation between temperature and spring constant. As temperature increased from 293 K to 353 K, the mean force required to extend the rubber band by 21 cm rose from 1.15 N to 1.45 N, corresponding to an increase in the average spring constant from 5.48 N m^{-1} to 6.90 N m^{-1}.

This trend supports the theoretical prediction derived from the entropic spring model,

$$k = (3K_b TN) / l^2,$$

where k represents the spring constant of the rubber band, T is the absolute temperature of the rubber band, N refers to the number of monomer units. and l is the length of each monomer segment

which indicates that the spring constant is directly proportional to the absolute temperature. The observed data thus confirms that as temperature increases, the entropic restoring force becomes stronger due to greater molecular motion within the polymer chains, resulting in a stiffer (higher k) rubber band.

The consistency of the increase, approximately 26% across a 60 K range, suggests high precision and reliability in the experiment. Despite minor random variations and one systematic deviation in trial 3, the overall data set shows a coherent linear relationship between k and T.

Therefore, the experimental results support the hypothesis that the spring constant of a rubber band is directly proportional to its temperature in the range of 293–353 K. This validates the entropic model of rubber elasticity within the limits of the experimental setup.